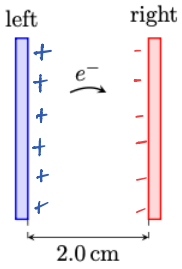
Problem 1. Two parallel, initially uncharged metal plates each have area $400\,\text{cm}^2$ and are separated by 2.0 cm (see below). A total of 1.0×10^{12} electrons are transferred from the left plate to the right plate, and the plates are then isolated. Treat the plates as large compared with their separation; the charge on each plate resides on its inner face (the face toward the other plate).

a. What is the surface charge density on the inner surface of each plate?

b. Find the magnitude and direction of the electric field between the plates.

c. Find the potential difference between the plates. Which plate is at the higher potential?



a) Charges will be attracted and reside on inside surfaces of the plates:

Left: $\sigma_L = \dfrac{(+1\times10^{12})(1.6\times10^{-19})}{0.04\,m^2} = +4\times10^{-6}\ C/m^2$

Right: $\sigma_R = -4\times10^{-6}\ C/m^2$

b) Between parallel plates: See lecture 6, page 6

$$\vec{E} = \frac{\sigma}{\epsilon_0} = \frac{4\times10^{-6}}{8.85\times10^{-12}}\uparrow = 4.52\times10^{5}\ V/m\uparrow$$

$$= 4.52\times10^{5}\ N/C\uparrow$$

to the right →

c) $\Delta V = Ed$

$= (4.52\times10^{5}\ V/m)(0.02)$

$= 9.04\times10^{3}$ Volts

Problem 2. In a certain region of space the electric potential is

$$V(x, y, z) = -xy^2z + 4xy,$$

where V is in volts and x, y, z are in metres (the implied constants carry the appropriate units).

a. Find the components E_x, E_y, E_z of the electric field.

b. Evaluate $\vec{E}$ (magnitude and direction, in terms of $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$) at the point $(1, 1, 1)$ m.

c. How much work must an external agent do to move a $+2.0\,\mu$C charge slowly from the origin to $(1, 1, 1)$ m? Does the answer depend on the path taken? Explain.

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}. \qquad \boxed{7.13}$$

2a) $V = -xy^2z + 4xy$

$E_x = -\frac{\partial V}{\partial x} = -(-y^2z + 4y) = y^2z - 4y$

$E_y = -\frac{\partial V}{\partial y} = -(-2xyz + 4x) = 2xyz - 4x$

$E_z = -\frac{\partial V}{\partial z} = -(-xy^2) = xy^2$

b) $\vec{E} = (y^2z - 4y)\hat{\imath} + (2xyz - 4x)\hat{\jmath} + xy^2(\hat{k})$

at $(1,1,1)$ $\vec{E}(1,1,1) = (-3\hat{\imath} - 2\hat{\jmath} + \hat{k})$ N/C

c) $V(0,0,0) = 0$

$V(1,1,1) = -3V$

$Work = Q\,\Delta V$

$= 2\times10^{-6}(3V - 0V)$

$= 6\times10^{-6}$ Joules

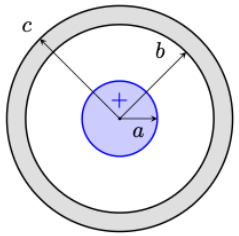
Problem 3. A solid conducting sphere of radius $a = 2.0\,\text{cm}$ carries charge $+5.0\,\mu\text{C}$. It is concentric with a thick conducting spherical shell of inner radius $b = 5.0\,\text{cm}$ and outer radius $c = 6.0\,\text{cm}$ that carries a net charge of $-3.0\,\mu\text{C}$ (see below).

a. Find the charge on the inner and outer surfaces of the shell.

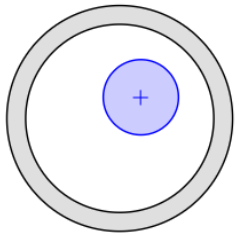
b. Use Gauss's law to find $E(r)$ in each of the four regions: inside the sphere ($0 \le r < a$), the gap ($a < r < b$), inside the shell ($b < r < c$), and outside ($r > c$).

c. Taking $V(\infty) = 0$, find the potential of the shell and of the inner sphere.

d. Sketch $E(r)$ and $V(r)$ on the same horizontal axis, $0 \le r \le 10\,\text{cm}$. Where is V continuous? Where is E discontinuous, and why?



e. Suppose the inner sphere is moved off-centre (without touching the shell), as shown below. Which of your electric-field answers to (b) change, and which do not? Sketch the electric field lines in all regions, showing how they meet each conducting surface.



the inner sphere has charge $+5\,\mu C$, this will attract a negative charge of $-5\,\mu C$ to the inner surface of the shell resulting in $q_{enc} = \emptyset$

$\therefore$ Charge on inner surface

$q_b = -5 \times 10^{-6}\,C$

3a) If we construct a Gaussian surface with radius $b < r < c$ it will be inside the conducting shell.

$E = \emptyset$ inside a conductor $\therefore$ for $b < r < c$ $E = \emptyset$ and $q_{enc} = \emptyset$

The total charge on the shell is

$$-3\mu C = q_b + q_c = -5\mu C + q_c$$

$$q_c = +2\mu C$$

b) Inside the sphere: The sphere is conducting (not uniformly charged)

So all charge resides on surface at $r = a$ for $0 \leq r < a$ $E = 0$

for $a < r < b$ Gauss's Law is:

$$E(4\pi r^2) = \frac{q_{enc}}{\epsilon_0} = \frac{+5\times10^{-6}\ C}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}(5\times10^{-6}\ C)\left(\frac{1}{r^2}\right) = \left(4.5\times10^{4}\ \frac{N m^2}{C}\right)\frac{1}{r^2}$$

(directed outward)

b) continued

Inside the shell $b < r < c$ $\quad E = 0$

Outside the shell $r > c$ $\quad q_{enc} = (5\,nC + (-3\,nC)) = +2\,nC$

$$E(4\pi r^2) = \frac{+2\,nC}{\epsilon_0} = \frac{2\times10^{-6}\,C}{8.85\times10^{-12}}$$

$$E = \left(+1.80\times10^{4}\,N\,m^2/C\right)\frac{1}{r^2}$$

(directed outwards)

c)

7.2 Electric Potential and Potential Difference

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ction

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Voltage and Electric Field

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}. \qquad 7.6$$

Consider the special case of a positive point charge q at the origin. To calculate the potential caused by q at a distance r from the origin relative to a reference of 0 at infinity (recall that we did the same for potential energy), let $P = r$ and $R = \infty$, with $d\vec{\mathbf{l}} = d\vec{\mathbf{r}} = \hat{\mathbf{r}}dr$ and use $\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$. When we evaluate the integral

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

for this system, we have

$$V_r = -\int_\infty^r \frac{k_e q}{r^2}\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}dr,$$

which simplifies to

$$V_r = -\int_\infty^r \frac{k_e q}{r^2}dr = \frac{k_e q}{r} - \frac{k_e q}{\infty} = \frac{k_e q}{r}.$$

This result,

$$V_r = \frac{k_e q}{r}$$

Following this approach from the textbook, we can work out the potential by integrating the electric field in from infinity...

For $r > c$ $\quad E = (+1.80 \times 10^4 \, N m^2/C)\frac{1}{r^2}$

$$V_r = -\int_\infty^r (E\hat{r}') \cdot (\hat{r}'dr') = -\int_\infty^r \frac{1.8\times10^4}{r'^2}dr' = 1.8\times10^4\left(\frac{1}{r} - 0\right)$$

$$= \left(\frac{+1.8\times10^4}{r}\right)$$

at $c = 6\times10^{-2}$ m $\quad V_c = \frac{1.8\times10^4}{(0.06)} = 3\times10^5$ V

From c to b the conductor is an equipotential, so the potential (voltage) stays constant at $V = 3 \times 10^5$ Volts

In the hollow region from $a < r < b$ we can find the potential difference by integrating E but to get the potential wrt infinity we must add $V_{(b)}$:

$$\Delta V_{br} = -\int_b^r \left(4.5\times10^4\ \mathrm{N\,m^2/C}\right)\frac{1}{r'^2}\,\hat{r}\cdot\left(\hat{r}\,dr'\right)$$

$$= 4.5\times10^4\left(\frac{1}{r'}\Big|_b^r\right)$$

$$= 4.5\times10^4\left(\frac{1}{r}-\frac{1}{b}\right)$$

at $r=0$ this gives $\Delta V_{br}=0$

we must add V_b to all these results to get correct potential wrt V_b:

$$V(r) = 3\times10^5 + 4.5\times10^4\left(\frac{1}{r}-\frac{1}{b}\right)$$

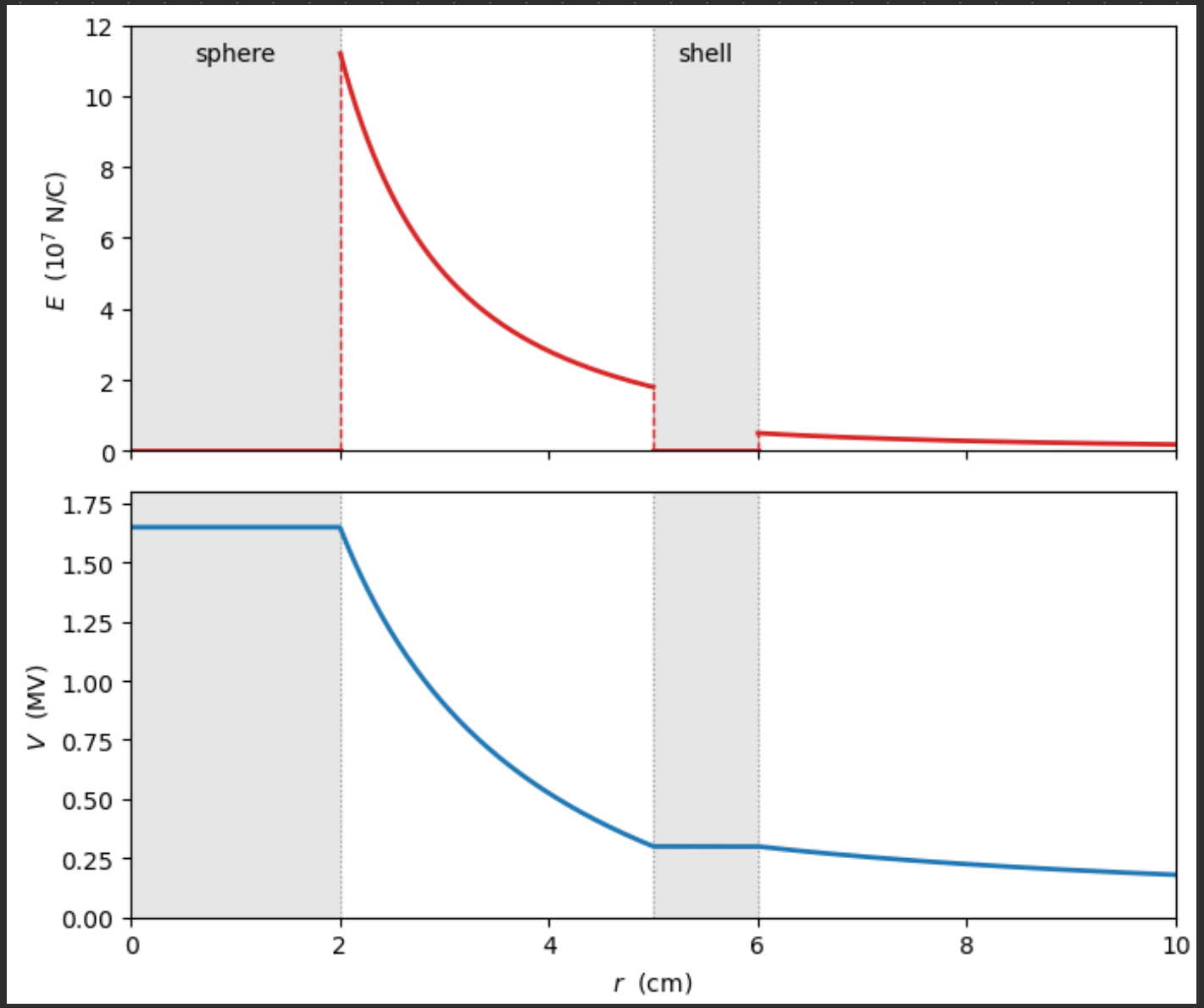
$a = 0.02\ m$, $b = 0.05\ m$

Now at $r = a$

$$V_{(a)} = 3 \times 10^5 + 4.5 \times 10^4 \left(\frac{1}{0.02} - \frac{1}{0.05} \right)$$

$$= 1.65 \times 10^6\ V$$

this potential is constant from $r = a$ to $r = 0$


sphere
shell
E (10^7 N/C)
V (MV)
r (cm)
0
2
4
6
8
10
12
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75