# PHYS 216 – Midterm #1 – Solutions and Marking Scheme

University of Victoria – Fall 2026 – Steeves

Duration: 50 minutes –

---

**Instructions:**

- **Answer all 4 problems in an exam booklet** (10 marks each, 40 marks total).
- You may answer the problems in any order.
- Show your work and clearly indicate final answers and units.
- Allowed: your formula sheet and a Sharp EL510 calculator.

$$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C}^2/(\mathrm{N}\cdot\mathrm{m}^2), \qquad k = 8.99 \times 10^9\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2, \qquad g = 9.81\,\mathrm{m/s}^2$$

$$\text{surface area of a sphere} = 4\pi r^2, \qquad \text{volume of a sphere} = \tfrac{4}{3}\pi r^3$$

**Integrals** (not all are needed; constants of integration omitted):

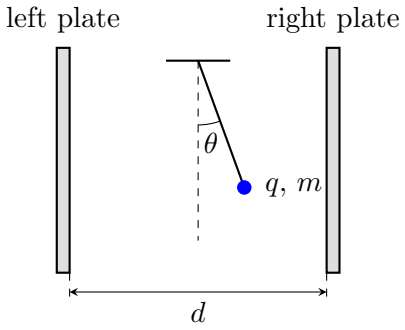
$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1), \qquad \int \frac{dx}{x} = \ln|x|$$

---

**Problem 1. [10 marks]** A small sphere of mass $m = 1.50\,\mathrm{g}$ and charge $q = +5.00 \times 10^{-7}\,\mathrm{C}$ hangs from a light thread between two large, vertical, parallel plates separated by $d = 4.00\,\mathrm{cm}$. A potential difference applied across the plates produces a uniform horizontal electric field, and in equilibrium the thread makes an angle $\theta = 20.0^\circ$ with the vertical, as shown.

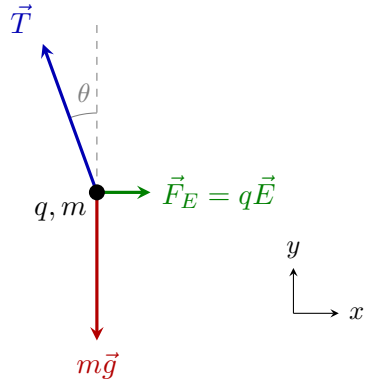
a. Draw a free-body diagram showing the forces acting on the sphere.

b. Find the magnitude of the electric field between the plates and the potential difference across the plates. Which plate is at the higher potential?



**Solution.**

a. Three forces: weight $m\vec{g}$ straight down; electric force $\vec{F}_E = q\vec{E}$ horizontal, toward the right plate; tension $\vec{T}$ along the thread toward the support. **[3]**




(Arrows drawn to scale: $T = 1.57 \times 10^{-2}\,\text{N}$, $mg = 1.47 \times 10^{-2}\,\text{N}$, $F_E = 5.36 \times 10^{-3}\,\text{N}$.)

b. *Field and potential difference* **[5]**
Equilibrium: $T\cos\theta = mg$ and $T\sin\theta = F_E$, so $F_E = mg\tan\theta$.
$F_E = (1.50 \times 10^{-3})(9.81)\tan 20.0^\circ = 5.36 \times 10^{-3}\,\text{N}$.
$E = \dfrac{F_E}{q} = \dfrac{5.36 \times 10^{-3}}{5.00 \times 10^{-7}} = 1.07 \times 10^4\,\text{N/C}$.
$\Delta V = Ed = (1.07 \times 10^4)(0.0400) = 428\,\text{V}$.

*Which plate is at the higher potential?* **[2]**
The left plate.
(Reason, not required: the positive sphere is pushed to the right, so $\vec{E}$ points from the left plate to the right plate, and $\vec{E}$ points from high to low potential.)

**Problem 2. [10 marks]** In a certain region of space the electric potential is

$$V(x, y, z) = 2x^2y - 3yz + 5z,$$

where $V$ is in volts and $x$, $y$, $z$ are in metres (the implied constants carry the appropriate units).

a. Find the components $E_x$, $E_y$, $E_z$ of the electric field and determine the magnitude and direction of the electric field at the origin.

b. How much work is required to move a $+2.0\,\mu\text{C}$ charge slowly from the origin to $(1, 1, 2)\,\text{m}$?

**Solution.**

a. $\vec{E} = -\nabla V$: **[6]**

$$E_x = -\frac{\partial V}{\partial x} = -4xy, \qquad E_y = -\frac{\partial V}{\partial y} = -2x^2 + 3z, \qquad E_z = -\frac{\partial V}{\partial z} = 3y - 5.$$

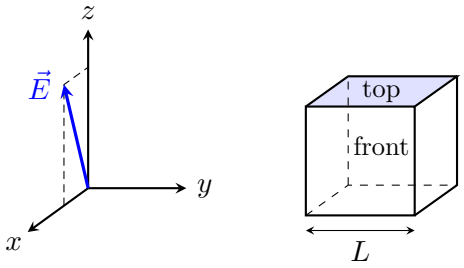
At the origin $E_x = 0$, $E_y = 0$, $E_z = -5$, so $\vec{E} = -5.0\,\hat{k}\ \text{V/m}$: magnitude $5.0\,\text{V/m}$, directed along $-z$.

b. $V(0,0,0) = 0$ and $V(1,1,2) = 2 - 6 + 10 = 6.0\,\text{V}$, so $\Delta V = 6.0\,\text{V}$. **[4]**
$W = q\,\Delta V = (2.0 \times 10^{-6})(6.0) = 1.2 \times 10^{-5}\,\text{J}$, independent of the path.

**Problem 3. [10 marks]** A cube of edge length $L = 0.20\,\text{m}$ has its faces parallel to the coordinate planes, as shown. It sits in a uniform electric field $\vec{E} = (2.0\,\hat{\imath} + 5.0\,\hat{k}) \times 10^3\,\text{N/C}$, which lies in the $xz$-plane.

a. Find the electric flux through the top face of the cube (outward normal $\hat{n} = +\hat{k}$) and through the bottom face (outward normal $\hat{n} = -\hat{k}$).

b. State the net flux through the whole cube, and briefly justify your answer.




The uniform field is now switched off and a point charge $q = +3.0\,\text{nC}$ is placed at the centre of the cube.

c. Use Gauss's law to find the net flux through the whole cube. Then, using symmetry, find the flux through the top face alone.

**Solution.** Each face has area $A = L^2 = (0.20)^2 = 0.040\,\text{m}^2$. For a uniform field through a flat face, $\Phi = \vec{E} \cdot \hat{n}\, A$.

a. $\Phi_{\text{top}} = \vec{E} \cdot (+\hat{k})\, A = E_z A = (5.0 \times 10^3)(0.040) = +2.0 \times 10^2\,\text{N m}^2/\text{C}$. **[4]**
$\Phi_{\text{bottom}} = \vec{E} \cdot (-\hat{k})\, A = -E_z A = -2.0 \times 10^2\,\text{N m}^2/\text{C}$ (flux enters through the bottom).

b. $\Phi_{\text{net}} = 0$. **[2]**
There is no charge inside the cube, so by Gauss's law the net flux is zero. (Equivalently: the top and bottom fluxes cancel, the front and back cancel in the same way, $\pm E_x A = \pm 80\,\text{N m}^2/\text{C}$, and $\vec{E}$ has no $y$-component, so the two side faces carry no flux.) Either argument is acceptable.

c. $\Phi_{\text{net}} = \dfrac{q}{\epsilon_0} = \dfrac{3.0 \times 10^{-9}}{8.85 \times 10^{-12}} = 3.4 \times 10^2\,\text{N m}^2/\text{C}$ (339). **[4]**
With the charge at the centre the six faces are equivalent, so they share the flux equally: $\Phi_{\text{top}} = \dfrac{q}{6\epsilon_0} = 56\,\text{N m}^2/\text{C}$ (56.5).

**Problem 4. [10 marks]** A solid non-conducting (insulating) sphere of radius $R$ carries a non-uniform charge density

$$\rho(r) = \rho_0 \frac{r}{R}, \qquad r \le R,$$

where $\rho_0$ is a positive constant. There is no charge outside the sphere.

a. Show that the total charge on the sphere is $Q = \pi \rho_0 R^3$.

b. Use Gauss's law to find the electric field $E(r)$ inside ($r < R$) and outside ($r > R$) the sphere.

**Solution.**

a. Divide the sphere into thin shells of volume $dV = 4\pi r^2\, dr$, so $dQ = \rho\, dV$. **[4]**

$$Q = \int_0^R \rho_0 \frac{r}{R}\, 4\pi r^2\, dr = \frac{4\pi\rho_0}{R} \cdot \frac{R^4}{4} = \pi \rho_0 R^3.$$

b. By spherical symmetry $\vec{E}$ is radial and has the same magnitude everywhere on a Gaussian sphere of radius $r$, so $E\,(4\pi r^2) = q_{\text{enc}}/\epsilon_0$. **[6]**

*Inside* ($r < R$): only part of the charge is enclosed,

$$q_{\text{enc}} = \int_0^r \rho_0 \frac{r'}{R}\, 4\pi r'^2\, dr' = \frac{\pi \rho_0 r^4}{R}, \qquad \text{so} \qquad E = \frac{\rho_0 r^2}{4\epsilon_0 R}.$$

*Outside* ($r > R$): $q_{\text{enc}} = Q$, so $E = \dfrac{Q}{4\pi\epsilon_0 r^2} = \dfrac{\rho_0 R^3}{4\epsilon_0 r^2}$.

In both regions $\vec{E}$ points radially outward. Check: at $r = R$ both expressions give $\rho_0 R/(4\epsilon_0)$.