PHYS 216 – Assignment #5 – Due Oct 28

Chapters 8–9: Dielectrics, Current & Resistance

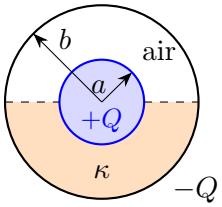
Problem 1. The spherical capacitor of Assignment 4, Problem 4 (inner sphere of radius a, thin outer shell of radius b, charges $\pm Q$) has capacitance $C_0 = 4\pi\varepsilon_0 ab/(b-a)$ when the space between the conductors is empty. Now the lower half of that space is filled with a liquid dielectric of dielectric constant κ, and the upper half is air (see below).

a. The field between the conductors is radial and has the same magnitude in both halves (you may assume this). Treat the arrangement as two hemispherical capacitors in parallel, one filled with dielectric and one with air, and find the capacitance C.

b. Find the potential difference between the conductors and the field $E(r)$ for $a < r < b$.

c. The free charge on the inner sphere is *not* uniformly distributed. Find the free surface charge density on the inner sphere in the air half and in the liquid half. What fraction of Q sits on the liquid half?

d. Find the bound surface charge density on the liquid where it touches the inner sphere. Show that the *total* (free plus bound) surface charge density is the same in both halves, and explain why this is consistent with the field being the same in both halves.



Problem 2. Two vertical parallel plates of width w (into the page) and separation d hang with their lower edges just touching the surface of a container of silicone oil (dielectric constant κ, density ρ). The plates are connected to a battery of potential difference V. The oil between the plates rises a height h above the level outside (measure h relative to the level with the battery disconnected, so surface tension can be ignored).

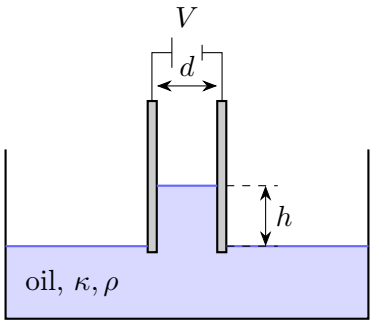
a. For plates of height H (from their lower edges, at the outside oil level), write the capacitance $C(h)$ as the sum of an oil-filled part and an air-filled part in parallel.

b. At constant V the upward force on the oil is $F = \frac{1}{2}V^2\, dC/dh$. Find F, and by balancing it against the weight of the raised oil show that

$$h = \frac{\varepsilon_0(\kappa - 1)V^2}{2\rho g d^2}.$$

c. Evaluate h for $\kappa = 2.7$, $\rho = 960\,\mathrm{kg/m^3}$, $d = 1.0\,\mathrm{mm}$ and $V = 2.5\,\mathrm{kV}$. The dielectric strengths of the oil and of air are $15\,\mathrm{MV/m}$ and $3.0\,\mathrm{MV/m}$. Is the capacitor safe from breakdown?

d. As the oil rises from 0 to h, the battery moves extra charge ΔQ onto the plates. Show that the work done by the battery, $V\Delta Q$, is twice the increase in stored energy. Where does the rest of the battery's energy go?



Problem 3. A manufacturing process produces hollow gold spheres with inner radius $a = 1.50\,\text{mm}$ and outer radius $b = 2.00\,\text{mm}$. The resistivity of gold is $\rho_0 = 2.44 \times 10^{-8}\,\Omega\,\text{m}$ at $T_0 = 20\,^\circ\text{C}$, with temperature coefficient $\alpha = 3.4 \times 10^{-3}\,^\circ\text{C}$.

a. Find a formula for the resistance between the inner and outer surfaces of a sphere at temperature T_0, and evaluate it. (Hint: add thin spherical shells of thickness dr in series.)

b. If the sphere is in an oven at $350\,^\circ\text{C}$, what is its resistance?

c. The same geometry appears in a grounding electrode at an electrical substation: a metal hemisphere of radius $a = 0.50\,\text{m}$ buried flat side up, flush with the surface, in soil of resistivity $\rho = 100\,\Omega\,\text{m}$. The soil now plays the role of the resistive material. The electrode is an ideal conductor ($0 \le r \le a$), so it holds the inner surface of the resistive "shell" at $r = a$ at a single potential, just as the inner surface did in (a). The distant earth plays the role of the outer surface, with $b \to \infty$. Adapt your formula from (a), accounting for the current spreading into only a half-space, to find the electrode's resistance to the distant earth.

d. During a fault, $I = 1.0\,\text{kA}$ flows into the ground through the electrode. The ground potential at distance r is $V(r) = I\rho/(2\pi r)$. A person stands with feet $1.0\,\text{m}$ apart along a radial line, at $r = 5.0\,\text{m}$ and $6.0\,\text{m}$. Find the "step voltage" between their feet.

