PHYS 216 – Assignment #4 – Due Oct 21

Chapters 7–8: Potential, Conductors & Capacitance

Problem 1. A Geiger tube consists of a thin central wire of radius $a = 0.10\,\text{mm}$ along the axis of a long conducting cylinder of inner radius $R = 2.00\,\text{cm}$ (see below). The wire is held at $V_0 = 900\,\text{V}$ above the cylinder. Treat the tube as infinitely long, with charge $+\lambda$ per unit length on the wire and $-\lambda$ per unit length on the inner surface of the cylinder.

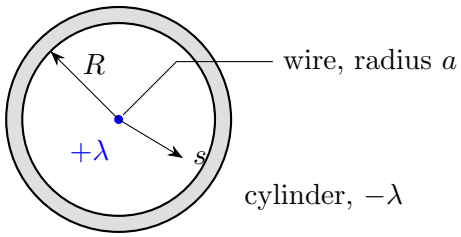
a. Use Gauss's law to find $E(s)$ for $a < s < R$, where s is the distance from the axis.

b. Find the potential difference $V_0 = V(a) - V(R)$ in terms of λ.

c. Eliminate λ to show that $E(s) = \dfrac{V_0}{s\,\ln(R/a)}$.

d. Dry air breaks down at about $3 \times 10^6\,\text{V/m}$. Explain why the very strong field is found only close to the wire, and why the tube is built with a thin wire rather than a thick rod.

e. From your result in (b), find the capacitance per unit length C/L of the tube in pF/m.



Problem 2. An isolated parallel-plate capacitor has plate area A, plate separation d, and charge $+Q$ on the bottom plate and $-Q$ on the top plate. An uncharged conducting slab of thickness $t < d$ and the same area A is slid between the plates, parallel to them (see below).

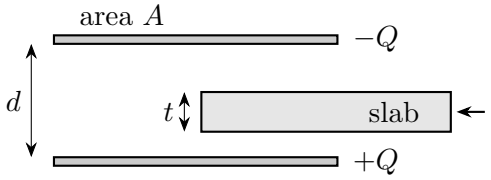
a. Find the capacitance before and after the slab is fully inserted. Does the vertical position of the slab between the plates matter?

b. Find the stored energy before and after. Does the energy increase or decrease?

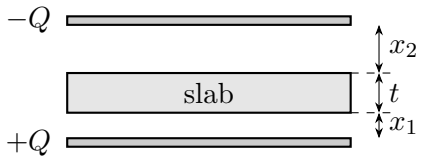
c. What is the surface charge density on the top and bottom faces of the slab once it is fully inserted?

d. Sketch the electric field lines when the slab is only partly inserted. Use your sketch to explain whether the slab is pulled into the capacitor or pushed out. As you slide the slab in slowly, do you do positive or negative work?

e. Now suppose the capacitor stays connected to a battery instead of being isolated. Does the stored energy increase or decrease as the slab goes in? Is the slab still pulled in? Account for the energy supplied by the battery.



(i) partly inserted



(ii) fully inserted

Problem 3. A $C_1 = 2.0\,\mu\text{F}$ capacitor and a $C_2 = 4.0\,\mu\text{F}$ capacitor are connected in series across a $1.00\,\text{kV}$ battery.

a. Find the charge on each capacitor, the potential difference across each, and the total stored energy.

b. The charged capacitors are disconnected from the battery and from each other, and then reconnected to each other in parallel, positive plate to positive plate and negative to negative. Find the final charge on and potential difference across each capacitor.

c. Find the stored energy after reconnection. What fraction of the original energy is lost, and where does it go?

Problem 4. A spherical capacitor consists of a conducting sphere of radius a carrying charge $+Q$, surrounded by a concentric thin conducting shell of radius b carrying $-Q$.

a. Find the potential difference between the conductors and hence the capacitance C.

b. Find the stored energy using $U = Q^2/2C$.

c. Now find the stored energy a second way: integrate the energy density $u_E = \frac{1}{2}\varepsilon_0 E^2$ over the volume between the conductors, using thin spherical shells of volume $4\pi r^2\,dr$. Confirm that you get the same answer as in (b).

d. Let $b \to \infty$ to find the energy of an isolated charged sphere. What fraction of this energy is stored within the region $a < r < 2a$? Within what radius is 90% of the energy stored?

e. A Van de Graaff dome is a sphere of radius $15\,\text{cm}$ held at $300\,\text{kV}$. Find its charge and the energy stored in its field.

